Simulation and Optimization of Class-E Power Amplifiers with Extended Impedance Method

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***Abstract*— This article introduces a novel perspective to the study of Class-E power amplifier by extending the scope of electrical impedance. Owing to this extension, the electrical property of the active switch in a Class-E circuit is taken as a special kind of impedance. Consequently, it is possible to regard a whole Class-E circuit as combination of impedances, whose total impedance is obtainable according to the existing circuit laws. When subjected to a DC supply voltage, the steady-state response of the total impedance can be directly obtained with the Ohm's law, regardless of the transient state. In addition, based on the formulation, regarding maximum power conversion efficiency as its design objective, Class-E optimization is also carried out. Since no waveform equation is required for both simulation and optimization, these simulation and optimization are more concise and efficient than those in all of the previous studies.**

I. Introduction

Firstly introduced in 1964 [1], the Class-E power amplifier (PA) is a kind of tuned switching DC-AC converters [2]. It is outstanding for its high conversion efficiency and good performance in high frequency and large power applications, e.g., output stages of radio transmitters [3], high-frequency electric process heating [4], and transcutaneous power and data links for implanted biomedical devices [5].

As shown in Fig. 1(a), the circuit of Class-E PA composes of six components. The presence of the switching component, i.e. the MOSFET in Fig. 1(a), adds difficulty to its analysis. In the previous studies, the switching component was modeled as an ideal switch and these six components were divided into three blocks: active switch, load network, and load [2], [6]. The block diagram is shown in Fig. 1(b).

The conversion efficiency of Class-E PA can be specified as

$$\eta = \frac{\bar{P}_{load}}{\bar{P}_{sup}}, \tag{1}$$

where $\bar{P}_{sup}$ and $\bar{P}_{load}$ are the average input power from the DC supply and the average power consumed by the load resistor R. Assuming all the components in the load network are ideal capacitors and inductors, the difference between $\bar{P}_{sup}$ and $\bar{P}_{load}$ is $\bar{P}_{sw}$, the power dissipated by the active switch. The efficiency η is maximized when this $\bar{P}_{sw}$ is minimized.

Sokal specified details of the conceptual "target" waveforms for the voltage across and current thought the switch [2], [6]. These specifications make sure that high voltage and high current do not exist at the same time, so that the switch dissipation can be minimized. Ever since then, most theoretical analyses started with waveform equations [1]–[8]; while most optimizations took the Class-E conditions, i.e. zero-voltage switching (ZVS) and zero-derivative switching (ZDS), which are also directly related to the waveforms, as standard criterions [1]–[7]. Yet, there are some insufficiencies with these analyses and optimizations. For analysis, the analytical waveforms may not be exactly the same as those in either circuit simulation or real situation; because waveform equations were usually derived based on some ideal assumptions. For optimization, satisfying ZVS and ZDS does not guarantee the highest efficiency; efficiency can be increased by moving away slightly from the nominal waveform [6].

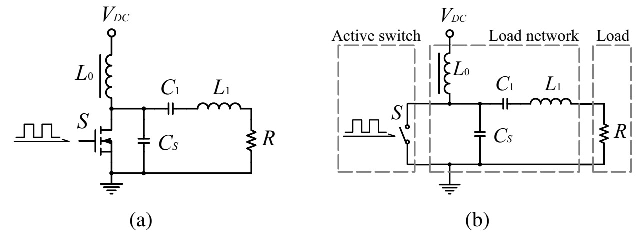

Fig. 1. Class-E power amplifier. (a) Circuit topology. (b) Block diagram used in previous studies.

In this article, an extended impedance method is proposed to analyze Class-E PA. It can serve for both Class-E PA simulation and optimization without any waveform equation involved. This analysis is more concise and straightforward than all previous studies.

II. Extended Impedance Method

The introduction of electrical impedance extended the Ohm's law to linear AC circuits and facilitated their analyses, which need to be carried out by solving differential equations before. In this section, this convenience is introduced to the analysis of Class-E PAs by extending the conceptual scope of impedance, so that the electrical properties of the active switching components in Class-E circuits can be incorporated as a special kind of impedance.

A. Time-Dependent Resistor

In Class-E circuit analysis, in order to consider the effect of a practical switching component, this component can be

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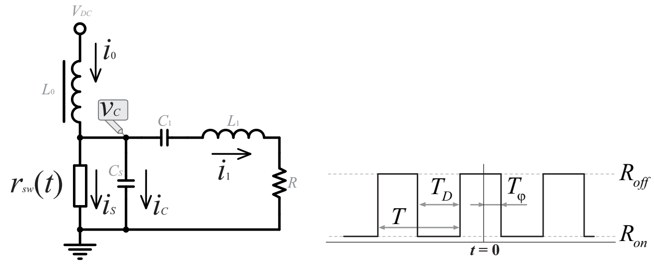


(a) (b)

Fig. 2. (a) Equivalent circuit of Class-E amplifier. (b) r_{sw} as a square wave function of time.

modeled as an ideal switch in series with the component's on-resistance [3], [7]. Further, Zhang et al. proposed the "time-dependent resistor model" to accurately simulate the power dissipation on the switch component [9]. Replacing the active switch with the time-dependent resistor r_{sw}, the Class-E equivalent circuit is shown in Fig. 2(a). The time-dependent characteristic of this r_{sw} is shown in Fig. 2(b). From the figure, the resistance value of r_{sw} switches between R_{on} and R_{off} with the period of T. The duty time, i.e. on-state interval, in every period is T_D, and initial phase is related to T_φ. In order to simplify the analysis, one of the switch-on instants is set to be time origin, i.e. let $T_\varphi = 0$. Thus, r_{sw} can be expressed as

$$r_{sw}(t) = \begin{cases} R_{on}, & nT \leqslant t < nT + T_D, \quad n \in \mathbb{Z}\,; \\ R_{off}, & nT + T_D \leqslant t < (n+1)T, \quad n \in \mathbb{Z}\,. \end{cases} \tag{2}$$

Note that, for every instant, r_{sw} is pure resistance, therefore, applying the Ohm's law, we can have the following relation

$$v_C(t) = r_{sw}(t)\, i_S(t)\,, \tag{3}$$

where $v_C(t)$ denotes the voltage across r_{sw} and $i_S(t)$ denotes the current flowing through r_{sw}.

Since the multiplication of two signals in time-domain corresponds to the convolution of their frequency representations in frequency-domain. The relation given by (3) now can be described in frequency-domain with

$$V_C(j\omega) = R_{sw}(j\omega) * I_S(j\omega)\,, \tag{4}$$

where $V_C(j\omega)$, $R_{sw}(j\omega)$ and $I_S(j\omega)$ are the counterparts of $v_C(t)$, $i_S(t)$ and $r_{sw}(t)$ in frequency-domain, respectively.

For periodic functions, their frequency expressions, i.e. Fourier transforms, are discretely combined with infinite impulses at their harmonic frequencies. For instance, the Fourier transform of periodic $v_C(t)$ is

$$V_C(j\omega) = \mathscr{F}[v_C(t)] = 2\pi \sum_{k=-\infty}^{\infty} V_{C,k}\, \delta(\omega - k\omega_0), \quad k \in \mathbb{Z}\,, \tag{5}$$

where $\omega_0 = 2\pi/T$ is the fundamental frequency, so $\delta(\omega - k\omega_0)$ denotes the impulse at the kth harmonic frequency, and $V_{C,k}$ is the kth Fourier coefficient of $v_C(t)$. Since $v_C(t)$ is a real function of time, the coefficients $V_{C,k}$ and $V_{C,-k}$ are a pair of complex conjugates. Likewise, the frequency representations of periodic $i_S(t)$ and $r_{sw}(t)$ can be obtained. Their kth Fourier coefficients are designated as $I_{S,k}$ and $R_{sw,k}$, respectively. In particular, for $r_{sw}(t)$ given in (2),

$$R_{sw,k} = \begin{cases} R_{on}D + R_{off}(1-D)\,, & k = 0, \\ \left(R_{on} - R_{off}\right)\dfrac{\sin(k\pi D)}{k\pi} e^{-jk\pi D}, & \mathit{others}, \end{cases} \tag{6}$$

where D is the duty cycle of the square wave driving signal.

With periodic $v_C(t)$, $i_S(t)$ and $r_{sw}(t)$, the convolution in (4) can be specified as the discrete convolution of their Fourier coefficients, i.e.

$$V_{C,k} = \sum_{n=-\infty}^{\infty} R_{sw,k-n}\, I_{S,n}\,, \tag{7}$$

or can be alternatively expressed in vector and matrix form:

$$\begin{bmatrix} \vdots \\ V_{C,-k} \\ \vdots \\ V_{C,0} \\ \vdots \\ V_{C,k} \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & R_{sw,0} & \ddots & R_{sw,-k} & \ddots & R_{sw,-2k} & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & R_{sw,k} & \ddots & R_{sw,0} & \ddots & R_{sw,-k} & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & R_{sw,2k} & \ddots & R_{sw,k} & \ddots & R_{sw,0} & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ I_{S,-k} \\ \vdots \\ I_{S,0} \\ \vdots \\ I_{S,k} \\ \vdots \end{bmatrix}. \tag{8}$$

Substituting the elements in (8) with the corresponding values given by (6), the frequency-domain expression of the electrical property of a certain time-dependant resistor can be obtained.

B. Impedance in Matrix Form

Assuming that most of the power concentrates at the harmonics whose orders are lower than a critical value of K, neglecting other harmonics out of this range, the periodic voltage $v_C(t)$ and current $i_S(t)$ can be represented in frequency-domain by vectors that build up by their *truncated Fourier series*, i.e.

$$\mathbf{V_C} = \begin{bmatrix} V_{C,-K} & \cdots & V_{C,-1} & V_{C,0} & V_{C,1} & \cdots & V_{C,K} \end{bmatrix}^{\mathrm{T}}, \tag{9}$$

$$\mathbf{I_S} = \begin{bmatrix} I_{S,-K} & \cdots & I_{S,-1} & I_{S,0} & I_{S,1} & \cdots & I_{S,K} \end{bmatrix}^{\mathrm{T}}, \tag{10}$$

where the superscript T denotes vector transpose. To maintain the multiplication relation, the matrix in (8) should also be truncated to be a square matrix with the dimension of $2K + 1$. Thereupon, the truncated version of (8) is

$$\mathbf{V_C} = \mathbf{Z_{sw} I_S}\,, \tag{11}$$

where $\mathbf{Z_{sw}}$ is the square matrix of

$$\mathbf{Z_{sw}} = \begin{bmatrix} R_{sw,0} & \ldots & R_{sw,-K} & \ldots & R_{sw,-2K} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ R_{sw,K} & \ddots & R_{sw,0} & \ddots & R_{sw,-K} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ R_{sw,2K} & \ldots & R_{sw,K} & \ldots & R_{sw,0} \end{bmatrix}. \tag{12}$$

Similarity can be observed by comparing (11) to the well-known Ohm's law. We define the matrix given by (12) as

TABLE I

CLASS-E CIRCUIT PARAMETERS USED IN SIMULATION*

Parameters	Values
Supply DC voltage (V_{DC})	24 V
Drive frequency (f_0)	4 MHz
Drive duty cycle (D)	50 %
Switch on-resistance (R_{on})	0.48 Ω
Switch off-resistance (R_{off})	100 kΩ
Choke inductance (L_0)	45.32 μH
Shunted capacitor (C_S)	1.83 nF
Load-series inductance (L_1)	0.91 μH
Load-series capacitor (C_1)	2.35 nF
Load resistance (R)	4.56 Ω
Load network Q ($\omega_0 L_1/R$)	5

*The parameters are generated by the software named "ClassE" [10], which is based on the revised design equations presented in [6].

the *impedance of time-dependent resistor*. The Ohm's law on ordinary impedance can also be expanded into truncated vector and matrix form. Different from $\mathbf{Z_{sw}}$, the truncated matrix of ordinary impedance is diagonal.

By extending the scope of impedance, expressions on the electrical properties of active switch and ordinary impedance are now unified. An unprecedented convenience has been brought to the analysis of Class-E PA.

III. SIMULATION OF CLASS-E PA

A. Circuit Description

With the definition in Section II and the traditionally established series and parallel laws, the whole Class-E equivalent circuit in Fig. 2(a) can be regarded as a combined impedance

$$\mathbf{Z_{Class-E}} = \mathbf{Z_{L0}} + \left(\mathbf{Z_{sw}^{-1}} + \mathbf{Z_{Cs}^{-1}} + \mathbf{Z_{load}^{-1}}\right)^{-1}, \quad (13)$$

in which

$$\mathbf{Z_{load}} = \mathbf{Z_{L1}} + \mathbf{Z_{C1}} + \mathbf{Z_{R}} . \quad (14)$$

In addition, according to (9), the vector expression of DC supply voltage V_{DC} can be constructed as

$$\mathbf{V_{DC}} = \Big[\quad \underbrace{0 \ \cdots \ 0}_{K} \quad V_{DC} \quad \underbrace{0 \ \cdots \ 0}_{K} \quad \Big]^{\mathrm{T}} . \quad (15)$$

This vector have only one nonzero element. The vector expression of the Class-E characteristic waveform v_C is

$$\mathbf{V_C} = (\mathbf{Y_{sw}} + \mathbf{Y_{C0}} + \mathbf{Y_{load}} + \mathbf{Y_{L0}})^{-1}\, \mathbf{Y_{L0}} \mathbf{V_{DC}} , \quad (16)$$

where $\mathbf{Y} = \mathbf{Z^{-1}}$ stands for admittance. The corresponding time-domain waveform of v_C can be obtained by doing Fourier series expansion, i.e.

$$v_C(t) \approx \left[e^{-jK\omega_0 t} \ \cdots \ e^{-j\omega_0 t} \ \ 1 \ \ e^{j\omega_0 t} \ \cdots \ e^{jK\omega_0 t} \right] \mathbf{V_C} . \quad (17)$$

Likewise, other characteristic current and voltage waveforms can be obtained with existing circuit laws.

Besides simulating the steady-state waveforms, the supplied power and load power at steady-state can also be calculated:

$$\begin{aligned} \bar{P}_{sup} &= \frac{1}{T}\int_T |V_{DC}(t)\, i_0(t)|\,\mathrm{d}t = \sum_{k=-\infty}^{+\infty} V_{DC,k}\, I_{0,k} \\ &= \mathbf{V_{DC}^{\mathrm{T}}} \mathbf{Y_{Class-E}} \mathbf{V_{DC}} ; \end{aligned} \quad (18)$$

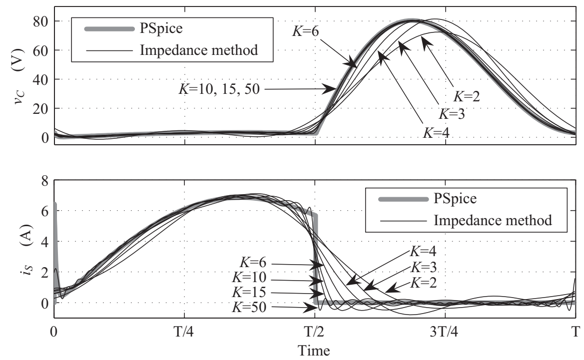


Fig. 3. Simulated v_C and i_S waveforms under different dimension K.

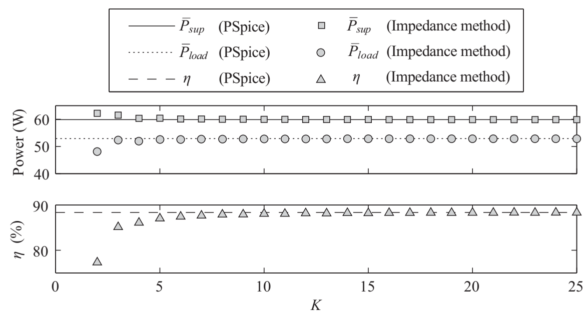


Fig. 4. Simulated power and efficiency under different dimension K.

$$\bar{P}_{load} = \mathbf{V_C^{\mathrm{T}}} \mathbf{Y_{load}} \mathbf{V_C} . \quad (19)$$

Both $\bar{P}_{sup}$ and $\bar{P}_{load}$ are scalars. Substituting these two powers into (1) yields the conversion efficiency η.

B. Steady-state Simulation

Previously, simulation of Class-E circuit was carried out by using simulation tools, e.g. PSpice. Steady-state waveforms could be obtained after getting through the transient-state. With the extended impedance approach, the steady-state waveforms of Class-E PA can be directly obtained with matrix manipulations, regardless of any duty cycle, finite RF choke, finite output network Q, any on-resistance, any parasitic resistances, whether tuned or untuned.

For instance, consider a 50% duty cycle tuned Class-E circuit, with its parameters given in Table I. The v_C and i_S waveforms are solved out using the extended impedance method under different dimension K. These waveforms are shown in Fig. 3, in comparison to the PSpice simulation result at 1ms (when waveforms are in steady-state) with a constant simulation step of 2.5×10^{-10} second. From these two sub-figures, the results with impedance method approach the PSpice one when K is getting larger. For v_C, when $K \geqslant 10$, the curves with impedance method almost overlap with the PSpice one; for i_S, K should be even larger, in order to involve more high harmonics to fit the steep current change at the instant of T/2.

Fig. 4 shows $\bar{P}_{sup}$, $\bar{P}_{load}$, and η as functions of K, respectively. They are very close to the PSpice result when $K > 10$.

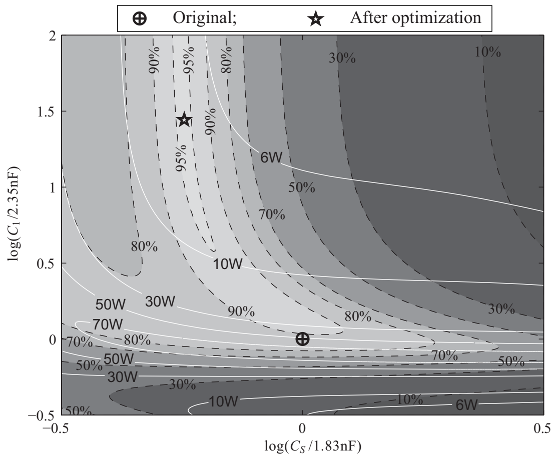


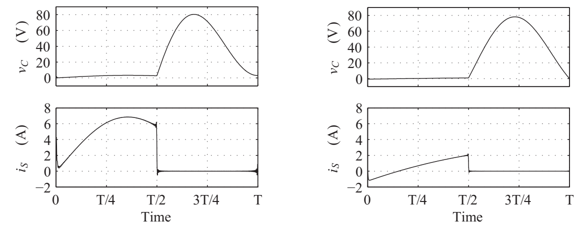
Fig. 5. Contours of η and $\bar{P}_{load}$ in the vicinity of the original values of C_S and C_1.

Since larger matrix dimension will require more computation, for discussion on power consumption and conversion efficiency, instead of waveform analysis, it is suggested that K should be set around 15 for this commonly used 50% duty cycle tuned Class-E PA, so as to meet the tradeoff between accuracy and computation time.

IV. Optimization of Class-E PA

The specification of ideal Class-E waveforms did intuitively explain the principle of achieving high efficiency; however, it confined the design focus on the so called Class-E conditions, i.e. ZVS and ZDS, which turned out to be insufficient to ensure the highest conversion efficiency [6]. In our study, since η can be obtained for both tuned or untuned circuits; taking η, rather than the Class-E conditions, as criterion, we can conduct Class-E PA optimization in a direct way. With this method, a Class-E PA can be optimized by adapting some of its component values.

Continue with the Class-E PA whose parameters were given in Table I. From Fig. 4, its efficiency is around 88%, which still could be further improved. Taking (1) as the objective function, while C_S and C_1 as free variables, better efficiency can be found with numeric searching method. In our study, a MATLAB program is employed to realize both steady-state simulation and numerical search for optimal efficiency. In the optimization, the termination tolerance of η is set to be 0.01% and the dimension K is set to be 25. After the numerical search, C_S stops at 1.05 nF and C_1 stops at 46.3 nF. Compared to the original circuit, η raises from 88.34% to 95.46%. Fig. 5 shows the contours of η and $\bar{P}_{load}$ in the vicinity of the original C_S and C_1, while two points representing the original and optimized C_S and C_1 pairs. Characteristic voltage and current waveforms before and after optimization are shown in Fig. 6. The optimized v_C satisfies the ZVS condition but does not satisfy the ZDS condition. The magnitudes of v_C before and after optimization are almost the same; whereas the magnitude of i_S after optimization is smaller than that before optimization, which implies that lower power can be



(a) (b)

Fig. 6. v_C and i_S waveforms. (a) Original. (b) After optimization.

delivered or converted after the optimization. With the new C_S and C_1, the power results of this optimized circuit can be calculated using the impedance method: $\bar{P}_{sup}$ = 7.10W and $\bar{P}_{load}$ = 6.78W. They are very close to the PSpice simulation results ($\bar{P}_{sup}$ = 7.12W, $\bar{P}_{load}$ = 6.81W, and η = 95.60%).

V. Conclusion

Unprecedented convenience has been brought to the analysis of Class-E power amplifiers (PAs) with the introduction of the extended impedance method proposed in this article. Rather than starting from waveform description, the simulation and optimization with this method were conducted in frequency-domain. They are concise, efficient, and compatible with traditional impedance based AC circuit analysis.

Acknowledgment

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