Revisit of synchronized electric charge extraction (SECE) in piezoelectric energy harvesting by using impedance modeling

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Abstract
Piezoelectric energy harvesting (PEH) systems convert ambient vibration energy into useful electricity. An interface circuit intervenes the electromechanical energy conversion; it has a significant effect on the electromechanical joint dynamics and harvested power. Among the existing interface circuits, the synchronized electric charge extraction (SECE) solution was known for its unique feature of load independence. However, the actual harvested power was usually shown to be lower than the previous theoretical predictions. The reason is that the energy dissipation in power conditioning, e.g. the diode dissipation in the rectifier and the switching dissipation in each energy extraction, have not received sufficient consideration. This paper revisits the joint dynamics and harvested power of PEH systems with a SECE interface circuit by using the energy flow analysis and impedance modeling. By qualitatively scrutinizing the energy cycle of SECE, the electrically induced dynamic characteristics are broken down into three components: the equivalent capacitance, dissipative resistance, and harvesting resistance, which have the same effects but different values, like those in other PEH interface circuits. The three components are equivalent to an additional stiffness, a dissipative damper, and a regenerative damper in the mechanical domain. The theoretical harvested power, which is estimated based on the impedance modeling, shows good agreement with the experimental results under different loading conditions and operating frequencies. Owing to its modular way of thinking, the impedance modeling technique once again shows its effectiveness and efficiency towards the analyses of joint dynamics and harvested power in PEH systems using different power conditioning interface circuits.

Keywords: Piezoelectric energy harvesting, synchronized charge extraction, electromechanical joint dynamics, impedance modeling, energy flow

(Some figures may appear in colour only in the online journal)

1. Introduction

Kinetic energy harvesting is one of the most extensively investigated energy harvesting technologies towards the realization of the energy-self-sufficient distributed Internet of Things (IoT) devices. The piezoelectric materials, as one of the favorite electromechanical transducers, can be used to construct compact (in mechanical structure) and efficient (in energy conversion) energy harvesting systems [1, 2]. Given these two outstanding features, during the last decade, much research effort has been made by mechanical engineers, electrical engineers, and material scientists towards better understanding and improvement of piezoelectric energy harvesting (PEH) systems.

The piezoelectric transducers generate AC (alternative current) voltage under vibration, while digital electronics as the load devices require a stable DC (direct current) voltage to run. A power conditioning circuit is necessary for fulfilling the
AC-to-DC power conversion. The studies of power conditioning interface circuits for PEH systems started from 2002, in which a simple full-wave bridge rectifier was used for the AC-to-DC conversion [3, 4]. Given its passive operation and easy-to-implement feature, the full-wave bridge rectifier is extensively used hereafter and referred to as the standard energy harvesting (SEH) interface circuit for PEH [5]. Later studies have shown that, besides only making the AC-to-DC conversion, the interface circuits have a significant effect on the PEH enhancement [2]. In particular, the PEH capability can be increased by several folds by using a family of power electronic circuits, which are called the synchronized switch harvesting on inductor (SSHI) [6, 7], or the bias-flip rectifier in the integrated circuit (IC) research community [8]. However, the harvested power by the aforementioned interface circuits varies under different loading conditions; a second stage maximum power point tracking (MPPT) module is necessary for maintaining the optimal harvested power for those solutions. Different from the SSHI solutions, the synchronous electric charge extraction (SECE) and its derivatives can also enhance the harvested power, which is independent of the load [9]. The load-independent feature of SECE is unique and always highlighted in the comparisons of different interface circuits [2, 6, 10]; even its maximum harvesting capability is not as significant as other synchronized switch counterparts. When the harvested power is independent of the loading condition, the second stage MPPT module is no longer necessary for tuning the equivalent load towards better harvesting performance. Therefore, the SECE solution is more compact and stable for practical PEH implementations.

The load-independence of SECE has attracted much research interest. It was proven that, under the weakly coupled condition, the harvested power obtained with SECE is approximately four times of that with SEH [10]. Wu et al. have developed a self-powered interface circuit called optimized SECE (OSECE) [11] for further increasing the harvested power beyond the four-time limitation under weak coupling. Lallart et al. have introduced a heterogeneous switching strategy by combining and coordinating the SSHI and SECE actions in different cycles and successfully got more harvested power under weak coupling. Considering the under-performance of SECE beyond mid-range electromechanical coupling, Richter et al. [12] and Xia et al. [13] have proposed the partial charge extraction solution for improving the harvesting capability of SECE under moderately and strongly coupled conditions. Morel et al. have realized another solution by heterogeneously skipping the switch actions in \( N \) cycles [14]. Such a solution was called N-SECE. The harvesting bandwidth problems with SECE have also been discussed and improved by introducing a phase lead/lag to the synchronized switch instants [15, 16]. Besides the new solutions towards better harvesting performance under different coupling levels and frequency bands, many other papers have discussed the implementation issues of SECE, e.g. the integrated circuit (IC) solutions of SECE [17, 18].

The interface circuits play an essential role in PEH enhancement under resonant conditions, in particular for weakly coupled systems [19, 20]. On the other hand, when putting the interface circuits into the practical electromechanically coupled scenario, the effect of the electrical manipulations, such as the electrically induced damping, can be quantified by studying the electromechanical joint dynamics of the PEH systems. Following the analytical formula proposed by Shu et al. in their analyses of SEH and SSHI [19, 20], Tang and Yang [10] have studied the joint dynamics and harvested power of SECE under different coupling conditions. As most of the studies about structural dynamics, the methodology used by Shu et al. starts from the equations of motion, the closed-form expressions summarizing the constitutive behavior of an entire system. In piezoelectric systems, such equations of motion are synthesized by the actuating equation, sensing equation, and constitutive relation of the nonlinear power conditioning circuit. The top-down solutions emphasize the system-level expression of the joint dynamics. The flattened and inclusive equation of motion might not be reusable when the interface circuit is changed. For example, to analyze the dynamics of PEH systems using SEH [19], parallel SSHI (P-SSHI) [20], series SSHI (S-SSHI) [21], or SECE [10], all derivations were redone from the beginning based on the energy-balance formula. Moreover, when a system gets more complicated, e.g. when the effect of dielectric loss must be included in the model [22], the top-down method can hardly lead to an explicit closed-form solution.

Like what a computer programmer encounters when writing a big and comprehensive computer program, the monolithic way of thinking makes it difficult to fulfill complicated or collaborative tasks. Therefore, a big program is often implemented by taking the modular way of thinking. By breaking down a big task into several independent modules, the final goal can be realized with better-defined procedures or sub-functions. Liang and Liao have proposed such a bottom-up modular solution for PEH systems in their impedance modeling and analysis for SEH, P-, and S-SSHI interface circuits [23]. In their study, by clarifying the energy flow within the PEH systems [5, 24], the dynamic behaviors of different interface circuits were decomposed into three general components, i.e. the harvesting resistance, dissipative resistance, and equivalent capacitance. The relations among those three equivalent components can be intuitively shown in the partitioned work cycles corresponding to different interface circuits. The use of different interface circuits does not change the system-level expressions but just differs in the values of the three equivalent components. Therefore, the system-level expressions can be reused; the impedance model can be easily transplanted to other PEH interface circuits with more comprehensive details [22] and even other kinetic energy harvesting systems [25].

In the study of a PEH system using the SECE interface circuit, Lefevre et al. have drawn the picture of the work cycle in their initial investigation [9]. The dynamic details have not been specified towards the general impedance expressions. Moreover, since the ideal lossless rectifier and lossless switch action were assumed in the previous studies, the observed experimental results are always lower than the simple theoretical prediction [9, 10]. Considering the aforementioned insufficiencies, this paper revisits the joint dynamics of the PEH systems using SECE and provides a more accurate theoretical prediction of the harvested power. A better insight towards the
2. System configuration

A typical PEH system is composed of three parts: the deforming mechanical substrate, piezoelectric transducer, and power conditioning circuit, as shown in figure 1. The interface circuit is at the front end of a power conditioning circuit converting the AC power from the piezoelectric transducer into a stable DC energy, which is stored in the energy storage. Given the bi-directional coupling effect of a piezoelectric transducer, the AC end of the interface circuit also introduces a backward dynamic effect to the vibrating structure. Therefore, the interface circuit plays a crucial role in PEH enhancement at resonance, and it also provides a possible way for broadening the bandwidth of the system [26, 27].

Given the constitutive relations in the piezoelectric elements [28], the electrical part of a PEH transducer can be regarded as a current source \( i_{eq} \), whose value is proportional to the vibration velocity \( \dot{x} \), connecting in parallel with the piezoelectric clamped capacitance \( C_p \) and the dielectric leakage resistance \( R_p \). The piezoelectric equivalent is shown in figure 2(a). \( R_p \) usually has a large value and is regarded as ignorable when the circuit does not operate under a high-power condition [22], e.g. with the SEH circuit or SECE in this paper; therefore, for the two currents shown in the piezoelectric equivalent, we have \( i_h \approx i_{eq} \). When no power conditioning circuit is connected, i.e. the open-circuit condition, the voltage across the piezoelectric transducer \( V_p \) is just proportional to the integral of \( i_h \). The magnitude of nominal open-circuit voltage

\[
V_{oc} = \frac{i_h}{\omega C_p} \approx \frac{i_{eq}}{\omega C_p},
\]

where \( I_h \) and \( I_{eq} \) are the magnitudes of \( i_h \) and \( i_{eq} \), respectively. \( \omega \) is the angular frequency of the vibration.

The SECE circuit operates at the open-circuit condition in most of a vibration cycle, except the zero-crossing instants of the equivalent current \( i_{eq} \), which are called the synchronized instants. At every synchronized instant, the initial condition of the \( i_{eq} \) integral is reset to zero by removing (extracting) all the charge stored in \( C_p \). The circuit operation and characteristic waveforms of the SECE interface circuit in a vibration cycle are shown in figure 2. The SECE circuit consists of a full-wave bridge rectifier and a buck-boost converter, which operates in a highly discontinuous conduction mode (DCM). But different from the conventional pulse width modulation (PWM) controlled buck-boost converter, the switching cycle of SECE strictly follows the mechanical vibrations at the synchronized instants, i.e. the zero-crossing points of velocity or, equivalently, the extremes of displacement.

In the positive-current half cycle, when \( i_{eq} > 0 \), the SECE action can be divided into three phases: the open-circuit, switching, and freewheeling (charging) phases, as illustrated in figures 2(a)–(i).

**Open-circuit phase.** Figure 2(a) illustrates the conducting paths during the open-circuit phase in red. The duration of this phase is approximately half of a vibration cycle. The switch \( S \) and diode \( D \) are at the off state; therefore, the SECE circuit takes no action during this phase. \( V_p \) is the integral of \( i_{eq} \) starting from a zero initial voltage, as highlighted by the red segment in figure 2(d) and (e).

**Switching (extraction) phase.** When the displacement of the piezoelectric beam reaches its maximum values, the switch \( S \) is turned on for slightly more than one fourth of an \( L_1/C_p \) cycle, in order to activate the switching phase. The conducting paths in the switch phase are highlighted in red in figure 2(b). In this phase, \( C_p \) and \( L_1 \) form an LC circuit, through which the energy stored in \( C_p \) is rapidly transferred to \( L_1 \). As \( V_p \), the voltage across the piezoelectric element is proportional to the charge stored in \( C_p \), \( v_p \) drops immediately to zero after one fourth of the \( L_1/C_p \) cycle, as highlighted in red in figure 2(f) and (g). Strictly speaking, \( v_p \) drops to twice of the diode forward voltage after one fourth of the \( L_1/C_p \) cycle. The turn-on interval of \( S \) is designed slightly longer than one fourth of the cycle to ensure the full discharge of \( C_p \). The charge extraction will be automatically stopped by the rectifier when \( v_p \) reaches zero. After this switch-on period, most of the energy stored in \( C_p \) is transferred into \( L_1 \), while the rest is dissipated at \( r \) the equivalent series resistance (ESR) of the switching branch.

**Freewheeling (charging) phase.** After the switch phase, the switch \( S \) is turned off, the energy accumulated in \( L_1 \) is transferred into the filter capacitor \( C_t \), through the freewheeling diode \( D \). \( C_t \) is designed to be much larger compared to \( C_p \) for providing a DC output voltage for the load resistance \( R \). The conducting paths in this phase are highlighted in red in figure 2(c). As the conducting paths are not connected to the piezoelectric source, they do not influence the piezoelectric element. In other words, the source and load are decoupled by the intermediate inductance \( L_2 \). Such a decoupling feature is quite similar to the buck-boost DC-to-DC converter in power electronics, except that SECE here converts AC power into DC. Since the piezoelectric element is not involved in the freewheeling phase, this phase has no effect on the piezoelectric voltage \( v_p \). The corresponding waveforms are shown in figure 2(h) and (i). The freewheeling diode \( D \) stops conduction until the current \( i_{eq} \) drops to zero, as shown in figure 2(i).

In the other half of a vibration cycle, i.e., the negative-current one when \( i_{eq} < 0 \), the operation can be also divided into three phases, which are reciprocal to those threes in the
positive-current half cycle. The corresponding conducting circuit branches and waveforms are shown in figure 2.

From the profile of piezoelectric voltage $v_p$ as shown in figure 2, we can find that it has the same sign as the current source $i_{eq}$ throughout a vibration cycle; therefore, the power extracted from the mechanical source is always positive. The analyses of SECE in the previous literature were based on the assumption of ideal lossless energy transfers from $C_p$ to $L_i$ and then from $L_i$ to $C_r$. Nevertheless, energy dissipation, in fact, exists in the power conditioning process. The energy dissipation is basically caused by:

- the practical bridge rectifier, whose forward voltage drop $V_F$ is nonzero;
- the practical series $L_iC_p$ circuit, whose quality factor $Q$ is finite;
- the practical freewheeling diode $D$, whose forward voltage drop $V_D$ is nonzero.

Such an energy dissipation mechanism might discount the total extracted energy and also affect the joint dynamics of the electromechanical system. The energy flow and equivalent dynamic details are also of necessity towards the comprehensive understanding of SECE.

3. Impedance of the $C_p$ and SECE combination

For evaluating the joint dynamics of an entire PEH system, we have to first formulate both the mechanical and electrical parts in a mathematical uniform. Considering the parameter-distributed feature of a piezoelectric cantilever and the nonlinear feature of a PEH power conditioning circuit, the most compatible way is to model or approximately express the dynamics of both parts in terms of equivalent impedance [22, 23].

In the equivalent impedance model, the mechanical part can be simply modeled as a single-degree-of-freedom (SDOF) lumped mass-spring-damper vibrator, whose vibration velocity $\dot{x}$ is proportional to the equivalent current $i_{eq}$ in the electrical part, as shown in figure 2, i.e.

$$i_{eq}(t) = \alpha_v \dot{x}(t),$$

where $\alpha_v$ is the voltage-to-force or velocity-to-current coupling factor. Considering the actual mechanical dynamics, the current source $i_{eq}$ in figure 2(a) represents the composition of a series resistor-inductor-capacitor (RLC) resonant circuit driven by an equivalent voltage $v_{eq}$, as shown in figure 3. The equivalent voltage is proportional to the force applied to the vibrator with the relation as follows

$$v_{eq}(t) = \frac{f(t)}{\alpha_e}.$$
The equivalent impedance network of a PEH device using the SECE interface circuit.

The equivalent resistor $R$, inductor $L$, and capacitor $C$ in the resonant circuit represent the damping $D$, mass $M$, and stiffness $K$ in the mechanical domain with the relations as follows [23]

$$R = \frac{D}{\omega_e^2}, \quad L = \frac{M}{\omega_e^2}, \quad C = \frac{\omega_e^2}{K}.$$  \hfill (4)

The SDOF electromechanical analogy of the mechanical vibrator was well established. The key problem to formulate the uniform model is how to express the equivalent impedance of the harvesting interface circuits. It was discussed in [22, 23] that the exact expression of the harvesting interface circuit cannot be obtained separately without considering the source impedance, in particular, the piezoelectric clamped capacitance $C_p$. Therefore, for the SECE circuit, we need to consider $C_p$ and the SECE circuit as a whole for quantifying the equivalent impedance.

Taking the equivalent impedance of the $C_p$ and SECE combination as $Z_{C_p\text{SECE}}$, we assume that the current flowing through $Z_{C_p\text{SECE}}$ as a sinusoidal current, i.e.

$$i_h(t) = I_h \sin(\omega t) \approx I_{eq} \sin(\omega t).$$  \hfill (5)

The approximation holds for large $R_p \gg \text{Re}[Z_{C_p\text{SECE}}]$, which might be violated when a power-boosting interface circuit, such as SSSH, is adopted [22]. In the experimental PEH structure used in this study, the ratio of $R_p$ over $\text{Re}[Z_{C_p\text{SECE}}]$ is about 5.6, as shown in figure 4, which satisfies the approximation in (5).

The SECE circuit operates at the open-circuit condition most of the time in a cycle, except the synchronized instants when $i_h$ crosses zero. At the synchronized instants, the SECE circuit resets the initial value of $v_p$, which is proportional to the integral of $i_h$ to zero. Given the waveforms shown in figure 2, $v_p(t)$ can be expressed with a piecewise equation, i.e.

$$v_p(t) = V_\infty \times \begin{cases} -\cos(\omega t) + 1, & 2k\pi \leq \omega t < (2k + 1)\pi, \\ -\cos(\omega t) - 1, & (2k + 1)\pi \leq \omega t < 2(2k + 1)\pi. \end{cases}$$  \hfill (6)

Given the $v_p$ expression in (6) and its waveforms shown in figures 2(d) and (f), the expression of the fundamental harmonic of $v_p(t)$ can be obtained as follows

$$v_{pf}(t) = V_\infty \left[ \frac{4}{\pi} \sin(\omega t) - \cos(\omega t) \right].$$  \hfill (7)

The waveform of $v_{pf}$ is illustrated in figures 2(d)–(i) and (m)–(r) by the dashed lines. The dynamics of $Z_{C_p\text{SECE}}$ can be obtained as the ratio of the fundamental harmonic voltage $v_{pf}$ over the harmonic current $i_h$ in the frequency domain, i.e.

$$Z_{C_p\text{SECE}}(j\omega) = \frac{V_{pf}(j\omega)}{I_h(j\omega)} = \frac{1}{\omega C_p} \left( \frac{4}{\pi} j - 1 \right).$$  \hfill (8)

For the three interface circuits discussed in [23], i.e. SEH, P-SSH, and S-SSH, the impedances of the $C_p$ and circuit combinations are tunable along the corresponding curves in the complex impedance plane, as shown in figure 4. The SECE is different from the aforementioned interface circuits in that it had a fixed impedance, as formulated in (8) and shown in figure 4 with the triangular marker. Moreover, the imaginary part of $Z_{C_p\text{SECE}}$ is equal to that in the open-circuit condition, i.e. $-j/(\omega C_p)$; its real part is $4/\pi$ times of the magnitude of the imaginary part. Unlike those circuits whose equivalent impedance is on specific one-dimensional trajectories. The zero-dimensional impedance point of SECE speaks its fixed dynamics and load independence in a PEH system. Also, under the weakly coupled condition, only the real part of $Z_{C_p\text{SECE}}$ decides the maximum extracted power of different interface circuits. As we can observe from figure 4, the SECE has enlarged the real part of the impedance, compared to SEH; yet, its real part cannot catch up with the SSSH interface circuits. Since the maximum real part of SEH is $1/\left(\pi \omega C_p\right)$ [23], the real part of SECE is just four times of this number in SEH. This four-time extracted power under the weakly coupled condition was proven in the previous literature [9, 10]. We here offer another explanation on this four-time relation from the equivalent impedance point of view.

With the harmonic approximation on the dynamics of the $C_p$ and SECE combination, the constitutive equation of the entire PEH system can be formulated with the basic circuit laws as follows

$$\frac{V_{eq}(j\omega)}{I_{eq}(j\omega)} = R + \frac{1}{j\omega C} + j\omega L + \frac{(4-j\pi)R_p}{4-j\pi+\pi\omega C_p R_p}.$$  \hfill (9)

It is a deterministic dynamic, which is not electrically tunable and only decided by the passive component values of the
piezoelectric structure. In other words, when the SECE interface circuit is used for power conditioning, the vibration of the PEH system does not change under different electrical loading conditions.

4. Detailed impedance breakdowns

With the impedance modeling, we have explained the well-understood result from another point of view. Besides, more insights on the dynamic details can be obtained by further breaking down the total impedance $Z_{C_p||SECE}$ into three compositions according to their different functional contributions towards the overall dynamics. The energy flow analysis is essential for revealing both the qualitative and quantitative relations among the three compositions [5, 24].

The general energy flow of an SECE-based PEH system is shown in figure 5(a). The mechanical energy enters the PEH system from the ambient vibration source and cycles between kinetic and potential energy in the mechanical resonance tank, as shown by the gray ring. In the resonant case, there is no energy return from the resonance tank to the source, while in the off-resonant cases, some of the energy flowing into the reactive component (equivalent mass or stiffness) returns back to the source. During the vibration, some of the mechanical energy is dissipated (converted into thermal energy) due to the mechanical damping, as shown by the orange branch in figure 5(a). The piezoelectric transducer transfers some mechanical energy into electricity. In SECE, since there is no energy return from the electrical part to the mechanical part, the converted energy is just the extracted energy from the mechanical part. This amount of extracted energy has two destinations in general. A part of it is converted into storable and useful electrical energy, which is called the harvested energy. The harvested energy is denoted with the green arrow in figure 5(a). The rest of the extracted energy is dissipated, i.e. converted into heat, during the power conditioning process. Such dissipation is caused by the aforementioned three reasons: the nonzero $V_F$ in the bridge rectifier, whose corresponding energy branch is denoted by the red arrow in figure 5(a); the parasitic resistance in the $L_p C_p$ loop, whose corresponding energy branch is denoted by

![Figure 5. Detailed energy compositions in SECE. (a) Energy flow chart. (b) The partitioned energy cycle.](image)
the blue arrow; and the nonzero \( V_D \) in the freewheeling path of the converter, whose corresponding energy branch is denoted by the purple arrow.

With the clarification on the detailed energy flow, we can have a more comprehensive insight on such a problem that the extracted energy is not equal to the harvested energy. The total extraction usually links with the loss factor, which evaluates the damping effect [5]. However, increasing the extracted energy does not guarantee better harvesting performance. The design should be done by taking the harvested energy or power as the target for optimization.

The quantitative analysis of extracted energy in SECE can be carried out by referring to the partitioned energy cycle as shown in figure 5(b). The energy is only extracted from the piezoelectric capacitance \( C_p \) to the harvesting circuit at the maximum or minimum \( V_p \) instants. Since \( V_p \) rapidly changes from \( 2V_{oc} \) or \( -2V_{oc} \) to zero, the extracted energy in each switching action, which happens in a half vibration cycle, can be obtained as follows

\[
\Delta E = 2C_pV_{oc}^2. \tag{10}
\]

During the switching process, the charge from \( C_p \) flows through the bridge rectifier with nonzero \( V_F \) and the inductive path with nonzero parasitic resistance \( r \). Both the practical components produce energy dissipation in each cycle, which can be respectively formulated as follows:

\[
E_{d,\text{rectifier}} = \Delta QV_F = 2C_pV_{oc}V_F, \tag{11}
\]

\[
E_{d,\text{switch}} = 2(1 + \gamma)C_pV_{oc}(V_{oc} - V_F), \tag{12}
\]

where \( \gamma \) is called the inversion factor in the SSHI solutions [5]. It is related to the quality factor \( Q \) of switching \( rL_rC_p \) circuit with the following relation

\[
\gamma = -e^{-\pi/(2Q)}. \tag{13}
\]

The switching dissipation was well-considered in SSHI. Yet, it has not got sufficient consideration in the previous SECE analysis, which leads to the overestimation on the actual available harvested power [9, 10]. For identifying the different energy portions in the extracted energy, the areas corresponding to \( E_{d,\text{rectifier}} \), \( E_{d,\text{switch}} \), and \( E_{d,\text{freewheeling}} \) are illustrated in red and blue in figure 5(b).

As shown in figure 6, at the end of the switching action, i.e. the \( t_2 \) instant, \( V_p \) arrives at zero and \( L_r \) the current flowing through the inductor attains its maximum value\(^1\) \( I_{L\text{,max}} \). The remaining energy, which is transferred into the inductor \( L_r \), can be formulated as follows

\[
E_{d,\text{max}} = 2|\gamma|C_pV_{oc}(V_{oc} - V_F). \tag{14}
\]

Different from the SSHI solutions, where the inductor is only used for voltage inversion, the inductor in SECE serves as an energy conveyor, which transports the energy from the piezoelectric source to the separated DC load.

After the \( t_2 \) instant, the freewheeling phase takes place. The energy stored in \( L_r \) is released to the storage capacitor \( C_r \) through the freewheeling diode \( D \). The current \( i_{L_d} \) decreases to zero with a constant slope of \(- (V_{DC} + V_D)\). The amounts of energy, which are absorbed by \( D \) and \( C_r \), are proportional to their corresponding DC voltages. The energy absorbed by freewheeling diode \( D \) is dissipated into heat, i.e.

\[
E_{d,\text{freewheeling}} = \frac{V_D}{V_{DC} + V_D}E_{d,\text{max}}. \tag{15}
\]

The area, which corresponds to \( E_{d,\text{freewheeling}} \), is illustrated in purple in figure 5. On the other hand, the energy absorbed by the DC source is the harvested energy, i.e.

\[
E_h = \frac{V_D}{V_{DC} + V_D}E_{d,\text{max}}. \tag{16}
\]

The area corresponding to the harvested energy is colored in green in figure 5.

The quantitative relations among \( E_{d,\text{rectifier}} \), \( E_{d,\text{switch}} \), \( E_{d,\text{freewheeling}} \), and \( E_h \) areas are shown in figure 5(b), given the ratios among (11), (12), (15), and (16). The total dissipated energy is obtained by combining the three parts of dissipated energy in the considered half vibration cycle, i.e.

\[
E_d = E_{d,\text{rectifier}} + E_{d,\text{switch}} + E_{d,\text{freewheeling}}. \tag{17}
\]

With the detailed breakdowns of the extracted energy, the real part of \( Z_{C_r}|_{\text{SECE}} \) can be quantitatively divided into two components, i.e. the harvesting resistance

\[
R_h = \frac{E_h}{\Delta E} \Re \left[ Z_{C_r}|_{\text{SECE}} \right] = 4 \frac{|\gamma|\tilde{V}_{DC}}{\omega C_p\tilde{V}_{DC} + \tilde{V}_D}(1 - \tilde{V}_F), \tag{18}
\]

and the dissipative resistance

\[
R_d = \frac{E_d}{\Delta E} \Re \left[ Z_{C_r}|_{\text{SECE}} \right] = 4 \frac{\tilde{V}_F + \left( 1 + \frac{\gamma\tilde{V}_{DC}}{\tilde{V}_{DC} + \tilde{V}_D} \right)(1 - \tilde{V}_F)}{\omega C_p}. \tag{19}
\]

where \( \tilde{V}_{DC} = V_{DC}/V_{oc} \), \( \tilde{V}_F = V_F/V_{oc} \), \( \tilde{V}_D = V_D/V_{oc} \) correspond to the normalized DC loading output voltage, normalized forward voltage drop of the rectifier, and normalized forward voltage drop of the freewheeling diode. \( R_h \) and \( R_d \) have the same damping effect on the vibrating system, yet their corresponding amounts of energy have different destinations, which should not be mixed up for PEH design and optimization.

\(^1\) The actual current of \( L_r \) at \( t_2 \) instant, when \( V_p = 0 \), is slightly smaller than the peak current.
As the values of all dynamic components in the equivalent impedance network shown in figure 3 are obtained, the harvested power can be calculated according to the basic circuit laws as follows [22]:

\[
P_h = \frac{V_{DC}^2 R_h}{2} \left[ \frac{R_p}{Z_{RLC} R_p + Z_{RLC} Z_{C_{p}}} + \frac{Z_{C_{p}}}{Z_{C_{p}}} R_p \right]^2
\]

\[
\approx \frac{1}{2} |Z_{RLC} + Z_{C_{p}}|^2,
\]

and the dissipated power is

\[
P_d = \frac{V_{DC}^2 R_d}{2} \left[ \frac{R_p}{Z_{RLC} R_p + Z_{RLC} Z_{C_{p}}} + \frac{Z_{C_{p}}}{Z_{C_{p}}} R_p \right]^2
\]

\[
\approx \frac{1}{2} |Z_{RLC} + Z_{C_{p}}|^2,
\]

where

\[
Z_{RLC} = R + j \left( \frac{\omega L}{\omega C} \right)
\]

is the total equivalent impedance of the mechanical part. All parameters in (20) are constants, except that \( R_h \) and \( R_d \) are functions of the normalized DC output voltage \( V_{DC} \). Therefore, strictly speaking, the harvested power also changes with the output characteristics in practical SECE systems. On the other hand, as we can observe from (18), the dependency is weakened when the voltage of the DC storage is much larger than the diode voltage drop (usually 0.5 to 0.7 volt). In other words, \( P_h \) approaches a constant power as \( V_{DC} \) gets larger. The harvested power is only a portion of the constant extracted power, rather than all of it. The impedance-based study provides more comprehensive insight on the performance of the practical SECE interface circuit.

5. Experimental validation

The harvested power analysis for the SECE interface circuit is validated by experiments, which are carried out on a base-excited PEH system. The experimental setup is shown in figure 7. The main structure is a piezoelectric cantilevered beam, which is composed of a PZT patch bonded on a copper cantilever. The cantilever is installed at a vibrating base, which is excited by a shaker. A pair of magnets is attached at the free end of the cantilever. The magnets also act as the proof mass for lowering the vibration frequency and increasing the deformation of the piezoelectric patches. An accelerometer, which is installed at the base, detects the acceleration in the vibration direction to provide a reference for controlling the vibration exciter. An electromagnetic coil installed near the magnet senses the relative speed of the cantilever beam. The output voltage is sent to the MSP430 micro-controller (EZ430-RF2500, Texas Instrument Inc.) for the synchronization purpose. Once the MCU catches a zero-crossing point of the sensed voltage, which is proportional to the vibration velocity, it immediately sends out a switch command to turn on the electronic switch in the SECE circuit. A multi-functional oscilloscope (DS1000Z, Rigol Technologies Inc.) is used as the
signal generator and oscilloscope. The magnitude of the base acceleration maintains at 8.6 m\(s^{-2}\) during the test by referring to the feedback from the accelerometer.

Before carrying out the power measurement, the internal parameters of the piezoelectric structure should be identified through curve fitting based on the data set obtained from an impedance analyzer (PSM3750, Newtons4th Ltd.). During the measurement, the base (fixed end of the beam) should be clamped still. The two electrodes of the piezoelectric transducer are connected to the impedance analyzer. Figure 8 shows the measured magnitude and phase of the impedance under different frequencies by discrete markers. The discrete data is fitted by estimating the component values based on the equivalent impedance model. The components values are later refined with a numerical algorithm. Both the magnitude and phase information are necessary for acquiring the accurate data set. In figure 8, both the magnitude (upper figure) and phase (lower figure) information are necessary for acquiring the accurate dynamic component values through the curve-fitting process. We measure the open-circuit voltage \(V_{eq}\) under a certain base acceleration \(Y\), such that we can derive \(V_{eq}\) magnitude based on the impedance network in figure 3. The coupling ratio \(\alpha_e\) can be derived indirectly according to the relation \(\alpha_e = V_{eq}/Y/L\). The final identified values of \(R, L, C, C_p, R_p, \) and \(\alpha_e\) are listed in table 1. Other specifications of the mechanical excitation and SECE circuit implementation are also listed in table 1.

The characteristic SECE voltage waveform in the experiment matches those in literature and the theoretical analysis shown in figure 2. Since the SECE waveform was shown in many papers; it is not repeated here. Assuming the PEH cantilever is excited by a base acceleration with constant magnitude, the harvested power under different vibration frequency and DC load \(R_l\) is calculated based on the formula provided in section 3. From the three-dimensional (3D) plotting shown in figure 9, two general changing trends can be observed. Under a specific vibration frequency, the harvested power rises from zero at small \(R_l\) and approaches a constant level at large \(R_l\). When the load \(R_l\) is fixed, the harvested power is maximized at the resonant frequency and decreases as the distance to the resonance getting larger. Therefore, the load-independent feature, in fact, is only valid when the vibration frequency is unchanged, and the load resistance is large.

In the experiment, the harvested power \(P_{h,exp}\) is obtained according to the measured DC output voltage \(V_{DC}\) across different connected \(R_l\) based on the following relation

\[
P_{h,exp} = \frac{V_{DC}^2}{R_l}.
\]

The theoretical and experimental results on harvested power \(P_h\) versus normalized DC output voltage \(\tilde{V}_{DC}\) under five vibration frequencies are shown in figure 10 for comparison. The theoretical prediction has a good agreement with the experimental results. The dashed lines in figure 10 show the extracted power under different vibration frequencies. The extracted power corresponds to the area enclosed by the work cycle trajectory, which was shown in figure 5(b). In the previous studies, the extracted power was simply taken the same as the harvested power. By identifying the harvested and dissipated portions from the total extracted power, the impedance modeling gives a more precise prediction on the harvested power as well as the dynamic contributions of different effects in the SECE-based PEH systems.

Figure 10(c) has shown the comparison between the harvested power in the SEH and SECE circuits under the 46 Hz resonant frequency. The theoretical extracted power in SECE is lower than four times (improvement in weak-coupling case) of the maximum harvested power in SEH, due to
the electrically induced damping effect, which reduces the deflection of the cantilever under the same base excitation. With the PEH systems in use, the actual maximum harvested power in SECE (under large $R_l$ or $\tilde{V}_{DC}$) is about 2.6 times of that in SEH, as shown in figure 10(c).

6. Discussion

It was known that the SECE circuit is sensitive to the alternative coupling coefficient $k^2Q_m$ [29], where

$$k^2 = \frac{\alpha_c^2}{KC_p} = \frac{C}{C_p}$$

is the original definition of the electromechanical coupling coefficient [28];

$$Q_m = \frac{\sqrt{KM}}{D} = \frac{1}{\omega_0} \sqrt{\frac{L}{C}}$$

is the quality factor of the mechanical resonator. By multiplying $k^2$ and $Q_m$ we can get

$$k^2Q_m = \frac{1}{\omega_0 C_p R}$$

where $\omega_0 = 1/\sqrt{LC}$ is the short-circuit resonant frequency. The sensitivity of the SECE solutions upon $k^2Q_m$ can be intuitively observed from the complex impedance plane.

For the weakly coupled case, i.e. $k^2Q_m \ll 1$, from (26), we can have $\omega_0 C_p R \gg 1$. The corresponding impedance picture is shown in figure 11(a). Figure 11(b) shows the harvested powers of ideal and practical SECE and SEH cases by normalizing them to the maximum attainable power under the impedance matching condition, i.e.

$$P_{max} = \frac{V_0^2}{8R}$$

From the impedance picture, we can see the blue rectangular marker, which represents the weakly coupling case, is much larger than the unity. The real part of neither SEH nor SECE can catch up with that of the source impedance. Both cases give small harvested power, ideally and practically, as shown by the curves in figure 11(b). On the other hand, the SECE real part is four times that of the real maximum in SEH; therefore, in the weakly coupled case, the harvested power ratio between maximums of the ideal lossless SECE and ideal SEH is also about four times, as shown by the blue dash-dot curve at the zero position. The power ratio of practical system at zero position will be higher, as the bridge rectifier cannot be conducted unless the open-circuit voltage is above the rectifier threshold.

When the system is moderately coupled, i.e. $k^2Q_m \approx 1$ or $\omega_0 C_p R \approx 1$ as shown by the purple rectangular marker in figure 11(a), the damping effect of SECE is comparable to the mechanical damping. From the impedance matching theory, maximum load power is obtained when the load resistance is
equal to the source resistance. Therefore, SECE attains the maximum harvestable power around unity \(k^2Q_m\), as shown by the purple dashed line in figure 11(b). More exactly speaking, the harvested power peaks at \(k^2Q_m = \pi/4\) if the circuit dissipation is omitted, i.e. ideal SECE. Since the maximum real part in SEH is \(1/\pi\) about one third of the unity, in the moderately coupled case, it cannot realize the impedance matching as the SECE does. However, the superior of SECE is weakened compared to that under the weakly coupled case. The ratio of SECE harvested power over the SEH maximum one is reduced and smaller than four. The experimental results in section 5 was obtained from a moderately coupled system, as \(k^2Q_m = 0.81\), which is close to the unity, according to table 1. The harvested power in ideal SECE crossover with that in ideal SEH at a second critical position is \(k^2Q_m = \pi/2\), as shown in figure 11(b). Practical systems might have a smaller power crossover critical coupling coefficient as the dissipation in SECE is more significant than that in SEH.

After the crossover, it enters the strongly coupled range, the rectangular marker further moves towards the zero along the real axis, as shown in figure 11, which makes \(k^2Q_m \gg 1\) or \(\omega_0C_pR \ll 1\). Since the equivalent impedance of the SEH circuit is tunable along the black dashed line in figure 11(a) [23], after a third critical point \(k^2Q_m = \pi\), there are two possible points for the ideal SEH to realize the impedance matching condition with the small source impedance \(R\). In contrast, SECE has a fixed equivalent impedance. It has no adaptability; therefore, it cannot realize the impedance matching under the strongly coupled condition. The harvestable power in SEH approaches the theoretical limit; while that in SECE decreases as the coupling gets stronger, as shown by the red dashed line in figure 11(b). In order to realize the impedance matching condition in the strongly coupled system. Researchers have developed some solutions, such as the partial discharge scheme [12, 13], to reduce the equivalent impedance magnitude by modifying the switch control for the SECE topology.

The aforementioned phenomenon under different coupling conditions were analyzed both theoretically and experimentally [9, 11]. The impedance modeling studied in this paper reinforces such an understanding in a more intuitive way by referring to the classical impedance matching theory.

Figure 11. Sensitivity of the maximum harvestable power over the alternative coupling coefficient \(k^2Q_m\). (a) Impedance picture. (b) Harvestable power versus \(k^2Q_m\).

7. Conclusion

The synchronized electric charge extraction (SECE) circuit was featured as the most investigated load-independent power conditioning interface circuits for piezoelectric energy harvesting (PEH) systems. This paper revisits the SECE and provides an improved analysis by using the impedance modeling. A more comprehensive understanding and power evaluation were obtained by dividing the extracted power into the harvested and dissipated portions, and visually showing their differences in the partitioned energy cycle. In particular, the dissipated power, which was not specified in the previous studies, was investigated in detail. Dissipation takes place because of the non-zero voltage drop of the practical rectifier, the finite quality factor of the inductive switching path, and the non-zero voltage drop of the freewheeling diode. The joint electromechanical dynamics is appropriately described by using the impedance modeling. The experimental results carried out on a cantilevered piezoelectric structure validate the theoretical analysis. It was more precisely revealed that the load-independent feature is valid when the vibration frequency is fixed, and the DC load resistance is large. Even the actual performance is not as ideal as it was assumed; the harvested power is still quite stable over a broad range of load resistance. Therefore, SECE is still unique and capable for building compact and robust PEH systems.

This case study on the SECE circuit once again showed the effectiveness and efficiency of the equivalent impedance modeling. Without changing the system-level framework and expressions, the joint dynamics and harvested power of a PEH system using an interface circuit can be easily formulated by adopting the expressions of the dissipative resistance, regenerative (harvesting) resistance, and equivalent capacitance with respect to the specific circuit. Such a modular way of thinking has brought much convenience to the analysis, design, and optimization of the holistic PEH systems.
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